

Some questions around the Pascal-adic transformation

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Adic transformations have been introduced by Vershik in the 1980's as combinatorial models for the cutting-and-stacking constructions in ergodic theory. Such a transformation acts on the space of infinite paths in a given Bratteli diagram. When this Bratteli diagram is the Pascal graph, we get the so-called Pascal-adic transformation, an example which is both very natural to consider, and highly non-trivial to study.

The family of invariant ergodic probability measures for the Pascal-adic transformation is simple to describe: it is parametrized by a real number p between 0 and 1, and the measure associated to p simply corresponds to drawing a random path in which each step is chosen to the right with probability p and to the left with probability $1-p$, independently of the other steps. Once such a measure is fixed, the Pascal-adic transformation gives rise to an ergodic, zero-entropy measure-theoretic dynamical system, whose properties are mostly unknown.

In this presentation, I intend to discuss some classical conjectures concerning this family of systems, in particular regarding their mixing properties and their classification in the zoo of zero-entropy measure-theoretic dynamical systems.