Markov partitions for Z^2-actions

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The following two properties of subshifts are well-known to be incompatible in one dimension:
- being of finite type (defined by a finite set of forbidden patterns),
- being aperiodic (non-empty, and all its configurations are non periodic).
Indeed, every non-empty 1-dimensional subshift of finite type contains a periodic configuration.

This is why hyperbolic automorphism of the 2-torus and irrational rotations on the circle are inherently different. Both admit a partition which gives a symbolic representation of the system. But, only the former admits a Markov partition, that is, a partition whose associated subshift is of finite type.
Irrational rotations of the circle can not admit a Markov partition since Sturmian subshifts are aperiodic.

These intuitions are wrong in dimension 2. We provide examples of Markov partitions for aperiodic Z^2-actions given by irrational rotations on the 2-torus. These examples are interesting since they use tools to study SFT that are typically used for Sturmian subshifts, for instance the Rauzy induction.

An open question is to characterize all Z^2-actions by rotations on the 2-torus which admit a Markov partition.